Conformal intercept of BFKL pomeron with NLO running coupling constant corrections

S.Bondarenko [a) *]

[a)] *University of Santiago de Compostela, Santiago de Compostela, Spain*

August 25, 2008

Abstract

In the present note we propose a shift of the anomalous dimension function of the eigenfunctions of the BFKL equation with the NLO running coupling corrections. The calculated eigenvalue of the modified equation turns out to be conformal invariant and we discuss consequences of this result.

1 Introduction

The approaches based on the BFKL equation, [1, 2], with the running coupling constant corrections included, [3, 4], were considered initially a long time ago. Some of these approaches were based on the including of the running coupling effects via the simple redefinition of the coupling constant into the running coupling constant in front of the equation, [5, 6, 7, 8, 11, 12, 13], some were based on the more complicated "bootstrap" ideas applied to the BFKL kernel, [9, 10]. The new direct calculations of the NLO kernel, show a coincide of the "bootstrap" approach with the results of diagram calculations, see [18, 17, 19], in the quark sector of the corrections, i.e. in the number of flavors n_{fl} leading order of the corrections. Still, whereas the "bootstrap" calculations include also a NLO gluonic part of the kernel the direct calculation of these contributions were completed only recently, see [14, 15, 16, 20]. In the present note we base our derivation on the base of "bootstrap" approach of the [9] and we do not consider a problem of the correctness of the obtained gluonic part of NLO BFKL kernel.

The common result of the considered earlier and present calculations is that the eigenvalue of the NLO BFKL kernel obtained with the use of the LO eigenfunctions is not conformal invariant anymore. The so called running coupling correction in the eigenvalue breaks the conformal invariance of the equation and, therefore, makes impossible to consider the NLO BFKL operator equation as the equation with properly found eigenfunctions and eigenvalue. Therefore, due the fact that the used eigenfunctions are not really eigenfunctions of the equation and that found eigenvalue is not really eigenvalue of the operator equation we face a problem of construction of the Green's function of the equation, see [12] for example. The NLO

*Email: sergey@fpaxp1.usc.es

Green's function, therefore, could be calculated only approximately with the use of different perturbative schemes in such a situation.

In the present note we propose a way to avoid this difficulty. We propose to modify the equation by including into the equation NLO corrections which arise from the perturbative expansion of the eigenfunctions. Practically it means that the anomalous dimension function of the eigenfunctions in the NLO approximation is different from the LO anomalous dimension function. Perturbative expansion of the NLO anomalous dimension function results in the redefinition of the eigenfunctions. The following perturbative expansion of the eigenfunctions leads to the new equation which has a conformal eigenvalue and, therefore, which could be used for the construction of the NLO Green's function.

The note is organized as follows. In the next section we remind LO and NLO results of the calculations of the BFKL operator equation, which we will use in the further derivations. In the Section 3 we redefine a anomalous dimesion of the LO eigenfunctions and obtain a new equation with conformal invariant eigenvalue. Section 4 dedicated to the Green's function of the new operator equation and Section 5 is a conclusion of the note.

2 LO and NLO BFKL kernel

First of all, let's remind, which kind of equation we consider. The BFKL equation, [1, 2], could be written as a operator equation with the BFKL kernel and corresponding eigenfunctions and eigenvalues

$$K \otimes \phi_f(k) = \omega_f \, \phi_f(k) \tag{1}$$

where ϕ_f is a eigenfunction and ω_f is a eigenvalue of the equation with kernel K, see [2, 5]. We begin from the LO BFKL kernel, [1, 2], and consider the equation at zero transferred momenta

$$\alpha_s \, K_{LO} \otimes \phi_\gamma(k) = \frac{N_c \, \alpha_s}{\pi^2} \int d^2 \kappa \left[\frac{2}{(\kappa - k)^2} \, \phi_\gamma(\kappa) - \frac{k^2}{\kappa^2 \, (\kappa - k)^2} \, \phi_\gamma(k) \right] \tag{2}$$

where all momenta k and κ must be understand as a two dimensional vectors. Due the non importance of that for the further derivations we do not underline this fact especially, introducing the vector notation only where it will be need. The eigenfunctions ϕ_f as a eigenfunctions of the operator equation must satisfy the completness relations

$$\sum_\gamma \phi_\gamma(k) \, \phi_\gamma(\kappa) = \delta^2 \, (k - \kappa) \tag{3}$$

and must be orthogonal each to other

$$\int d^2 k \, \phi_\gamma(k) \, \phi_{\gamma'}(k) = \delta \, (\gamma - \gamma') \tag{4}$$

The form of these eigenfunction is well known, it is

$$\phi_\gamma(k) \propto \left(\frac{k^2}{\mu^2} \right)^\gamma \tag{5}$$

Here we omitted a normalization factor in front of the function and we changed a usual definition of the eigenfunction introducing some external scale μ into expression. It is clear, that this scale is cancelled in the usual LO BFKL equation, leading only to the redefinition of the Green's function of the equation.

Defined with the new eigenfunctions the Green's function will be dimensionless instead the k^{-2} dimension of the Green's function defined with the use of usual k^{γ} eigenfunctions. The eigenvalue of the equation,

$$\omega_{\gamma}^{LO} \,=\, \frac{N_c\,\alpha_s}{\pi}\,\left(\,2\,\psi(1)\,-\,\psi(-\gamma)\,-\,\psi(1+\gamma)\,\right)\,=\,\frac{N_c\,\alpha_s}{\pi}\,\chi(-\gamma) \tag{6}$$

calculated with the help of this eigenfunctions, is the LO intercept of the BFKL Pomeron.

The NLO correction of the kernel, which are arising due the running coupling effect, were established a long time ago on the basis of the bootsrap conditions applied to the kernel, see [9, 10]. The rule, found for the introduction of the corrections in the [9], is very simple. Instead the LO propagator the following propagator must be used in BFKL equation at NLO

$$\frac{\alpha_s}{k^2}\,\rightarrow\,\frac{\alpha_s(k^2)}{k^2}\,=\,\frac{\alpha_s}{1\,+\,\frac{\beta_0\,\alpha_s}{4\,\pi}\,ln(k^2/\mu^2)}\frac{1}{k^2}\,=\,\alpha_s\left(\frac{1}{k^2}\,-\,\frac{\beta_0\,\alpha_s}{4\,\pi}\,ln(k^2/\mu^2)\frac{1}{k^2}\right) \tag{7}$$

where as usual $\beta_0\,=\,\frac{11\,N_c}{3}\,-\,\frac{2}{3}\,n_{fl}$ and as a renormalization scale we took the same μ^2 as in Eq.5 So, the NLO running coupling corrections determine the following form NLO kernel

$$\alpha_s^2\,K_{NLO}\otimes\phi_{\gamma}(k)=-\frac{N_c\alpha_s^2\beta_0}{4\pi^3}\int d^2\kappa\Big[\frac{2}{(\kappa-k)^2}ln\left(\frac{(\kappa-k)^2}{\mu^2}\right)\phi_{\gamma}(\kappa)-\frac{k^2}{\kappa^2\,(\kappa-k)^2}ln\left(\frac{\kappa^2\,(\kappa-k)^2}{k^2\mu^2}\right)\phi_{\gamma}(k)\Big] \tag{8}$$

We see, that obtained expression is coincide with the direct calculations of [17, 19] for the leading n_{fl} order. Now, using Eq.5 eigenfunctions and methods of the calculation of [21, 17] we have

$$\frac{N_c\alpha_s^2\beta_0}{4\pi^3}\int d^2\kappa\left[\frac{1}{(\kappa-k)^2}ln\left(\frac{(\kappa-k)^2}{\mu^2}\right)\left(\frac{\kappa}{\mu}\right)^{2\gamma}-\frac{k^2}{\kappa^2\,(\kappa-k)^2}ln\left(\frac{\kappa^2\,(\kappa-k)^2}{k^2\mu^2}\right)\left(\frac{k}{\mu}\right)^{2\gamma}\right]= \tag{9}$$

$$=\frac{N_c\,\alpha_s^2\,\beta_0}{2\,\pi^2}\Big[2\,\psi^2(1)+2\,\psi(1)\,ln\left(\frac{k^2}{\mu^2}\right)\,+\,2\psi(1)\frac{\Gamma(1+\gamma)}{\Gamma(-\gamma)}\frac{\partial}{\partial\,\gamma}\frac{\Gamma(-\gamma)}{\Gamma(1+\gamma)}$$

$$+\,ln\left(\frac{k^2}{\mu^2}\right)\,\frac{\Gamma(1+\gamma)}{\Gamma(-\gamma)}\frac{\partial}{\partial\,\gamma}\frac{\Gamma(-\gamma)}{\Gamma(1+\gamma)}\,+\,\frac{1}{2}\frac{\Gamma(1+\gamma)}{\Gamma(-\gamma)}\frac{\partial^2}{\partial\,\gamma^2}\frac{\Gamma(-\gamma)}{\Gamma(1+\gamma)}\Big]\,\left(\frac{k}{\mu}\right)^{2\gamma}\,=$$

$$=\,\frac{N_c\,\alpha_s^2\,\beta_0}{2\,\pi^2}\left(ln\left(\frac{k^2}{\mu^2}\right)\,\chi(-\gamma)\,+\,\frac{1}{2}\chi^2(-\gamma)\,+\,\frac{1}{2}\,\psi^{'}(-\gamma)\,-\,\frac{1}{2}\,\psi^{'}(1+\gamma)\right)\,\left(\frac{k}{\mu}\right)^{2\gamma}$$

Summing up all terms together we obtain

$$\alpha_s\,K_{LO}\,\otimes\,\phi_{\gamma}(k)\,+\,\alpha_s^2\,K_{NLO}\otimes\,\phi_{\gamma}(k)=\frac{N_c\,\alpha_s}{\pi}\Big[\chi(-\gamma)\,\left(1\,-\,\frac{\alpha_s\,\beta_0}{2\,\pi}\,ln\left(\frac{k^2}{\mu^2}\right)\right)\,- \tag{10}$$

$$-\,\frac{\alpha_s\,\beta_0}{2\,\pi}\,\left(\frac{1}{2}\chi^2(-\gamma)\,+\,\frac{1}{2}\,\psi^{'}(-\gamma)\,-\,\frac{1}{2}\,\psi^{'}(1+\gamma)\right)\Big]\,\phi_{\gamma}(k)$$

All these results are well known, see [17] for example. The only reason to reproduce these calculations is the expression Eq.10. Clearely, in spite of the Eq.6 the expression in the brackets in the r.h.s. of Eq.9 is not eigenvalue of the BFKL equation. The $ln\left(\frac{k^2}{\mu^2}\right)$ term, breaking conformal invariance of the expression, makes impossible to interpetate the expression as the eigenvalue and, correspondingly, as the intercept of the BFKL Pomeron. We see, that the functions Eq.5 are not eigenfunctions of the NLO BFKL kernel.

3 Conformal intercept

Now we come back to the Eq.10 and will shift the anomalous dimension of the eigenfunction in this expression

$$\gamma \, \rightarrow \, \gamma \, + \, \alpha_s \, \gamma_1 \, = \, f \tag{11}$$

In this case we have instead Eq.10

$$\Big(K \, \otimes \, \phi_f(k) \, = \, \alpha_s \, K_{LO} \, \otimes \, + \alpha_s^2 \, K_{NLO} \Big) \, \otimes \, \phi_{\gamma \, + \alpha_s \, \gamma_1} \, (k) \, = \tag{12}$$

$$= \, \frac{N_c \, \alpha_s}{\pi} \, \Big[\chi(- \gamma \, - \, \alpha_s \, \gamma_1 \,) \, \Big(1 \, - \, \frac{\alpha_s \, \beta_0}{2 \, \pi} \, ln \left(\frac{k^2}{\mu^2} \right) \Big) \, -$$

$$- \, \frac{\alpha_s \, \beta_0}{2 \, \pi} \, \Big(\frac{1}{2} \chi^2(- \gamma) \, + \, \frac{1}{2} \, \psi^{'}(- \gamma) \, - \, \frac{1}{2} \, \psi^{'}(1 + \gamma) \Big) \Big] \, \phi_{\gamma \, + \alpha_s \, \gamma_1} \, (k)$$

where we cared only about α_s^2 order terms. In order to continue further derivation let's assume that the following expansion of the anomalous dimension function of the eigenfunction holds

$$f \, = \, \gamma \, + \, \sum_{m=1} \, \alpha_s^m \, \gamma_m \tag{13}$$

and, therefore, the perturbative expansion of the $\phi_f(k)$ functions over the complete set of initial $\phi_\gamma \, (k)$ eigenfunctions in this case will have the following form

$$\phi_f(k) \, = \, \left(\frac{k^2}{\mu^2} \right)^f \, = \sum_{n=1} \, \alpha_s^{n-1} \, \gamma_{n-1} \, \left(ln \left(\frac{k^2}{\mu^2} \right) \right)^{n-1} \, \left(\frac{k^2}{\mu^2} \right)^\gamma \tag{14}$$

with $\gamma_0 \, = \, 1$. Keeping in this expansion only α_s order terms and incerting it back into the Eq. (12) we obtain

$$K \, \otimes \, \phi_f(k) \, = \, \Big[\alpha_s \, K_{LO} \, + \, \alpha_s^2 \, K_{NLO} \Big] \, \otimes \, \left(1 \, + \, \alpha_s \, \gamma_1 \, ln \left(\frac{k^2}{\mu^2} \right) \right) \, \left(\frac{k^2}{\mu^2} \right)^\gamma \, = \tag{15}$$

$$= \, \Big[\alpha_s \, K_{LO} \, + \, \alpha_s^2 \, \tilde{K}_{NLO} \Big] \, \otimes \, \left(\frac{k^2}{\mu^2} \right)^\gamma \, = \, \frac{N_c \, \alpha_s}{\pi} \, \Big[\chi(- \gamma \, - \, \alpha_s \, \gamma_1 \,) \, \Big(1 \, - \, \frac{\alpha_s \, \beta_0}{2 \, \pi} \, ln \left(\frac{k^2}{\mu^2} \right) \Big) \, -$$

$$- \, \frac{\alpha_s \, \beta_0}{2 \, \pi} \, \Big(\frac{1}{2} \chi^2(- \gamma) \, + \, \frac{1}{2} \, \psi^{'}(- \gamma) \, - \, \frac{1}{2} \, \psi^{'}(1 + \gamma) \Big) \Big] \, \phi_{\gamma \, + \alpha_s \, \gamma_1} \, (k) \, =$$

$$= \, \frac{N_c \, \alpha_s}{\pi} \, \Big[\chi(- \gamma) \, - \, \frac{\alpha_s \, \beta_0}{2 \, \pi} \, \Big(\frac{1}{2} \chi^2(- \gamma) \, - \, \frac{1}{2} \, \psi^{'}(- \gamma) \, + \, \frac{1}{2} \, \psi^{'}(1 + \gamma) \Big) \Big] \, \phi_{\, \gamma} \, (k)$$

where we used

$$\gamma_1 \, = \, \frac{\beta_0}{4 \, \pi} \tag{16}$$

and where we modified K_{NLO} kernel adding to it corrections which arise from $ln \left(\frac{k^2}{\mu^2} \right)$ correction of the eigenfunction

$$\tilde{K}_{NLO} \otimes \left(\frac{k^2}{\mu^2} \right)^\gamma \, = \, - \frac{N_c \alpha_s^2 \beta_0}{4 \pi^3} \int \, d^2 \kappa \Big[\frac{2}{(\kappa - k)^2} ln \left(\frac{(\kappa - k)^2}{\kappa^2} \right) \phi_\gamma(\kappa) - \frac{k^2}{\kappa^2 \, (\kappa - k)^2} ln \left(\frac{\kappa^2 \, (\kappa - k)^2}{k^4} \right) \phi_\gamma(k) \Big] \tag{17}$$

So, using Eq. (11) shift of the anomalous dimension of the eigenfunction, we obtain for the NLO BFKL equation

$$\Big[\alpha_s \, K_{LO} \, + \, \alpha_s^2 \, \tilde{K}_{NLO} \Big] \, \otimes \, \left(\frac{k^2}{\mu^2} \right)^\gamma \, = \, \left(\omega_\gamma^{LO} \, + \, \omega_\gamma^{NLO} \right) \, \left(\frac{k^2}{\mu^2} \right)^\gamma \tag{18}$$

where

$$\omega_{\gamma}^{NLO}\,=\,-\,\frac{N_c\alpha_s^2\,\beta_0}{2\,\pi^2}\,\left(\frac{1}{2}\chi^2(-\gamma)\,-\,\frac{1}{2}\,\psi^{'}(-\gamma)\,+\frac{1}{2}\,\psi^{'}(1+\gamma)\right) \tag{19}$$

is the NLO conformal intercept of BFKL equation.

Definitely the same answer we obtain if we will calculate the correction to the K_{NLO} kernel. Calculating the following integral

$$\alpha_s^2\gamma_1 K_{LO}\otimes ln\left(\frac{k^2}{\mu^2}\right)\left(\frac{k}{\mu}\right)^{2\gamma}\,=\,\frac{N_c\,\alpha_s^2\,\gamma_1}{\pi^2}\int d^2\kappa\Big[\frac{1}{(\kappa-k)^2}ln\left(\frac{\kappa^2}{\mu^2}\right)\left(\frac{\kappa}{\mu}\right)^{2\gamma}\,-\,\frac{k^2}{\kappa^2\,(\kappa-k)^2}ln\left(\frac{k^2}{\mu^2}\right)\left(\frac{k}{\mu}\right)^{2\gamma}\Big] \tag{20}$$

which gives

$$\frac{N_c\,\alpha_s^2\,\gamma_1}{\pi^2}\,\int d^2\kappa\left[\frac{1}{(\kappa-k)^2}ln\left(\frac{\kappa^2}{\mu^2}\right)\left(\frac{\kappa}{\mu}\right)^{2\gamma}\,-\,\frac{k^2}{\kappa^2\,(\kappa-k)^2}ln\left(\frac{k^2}{\mu^2}\right)\left(\frac{k}{\mu}\right)^{2\gamma}\right]\,= \tag{21}$$

$$=\,\frac{2\,N_c\,\alpha_s^2\,\gamma_1}{\pi}\Big[2\,\psi(1)\,ln\left(\frac{k^2}{\mu^2}\right)\,+\,ln\left(\frac{k^2}{\mu^2}\right)\,\frac{\Gamma(1+\gamma)}{\Gamma(-\gamma)}\,\frac{\partial}{\partial\,\gamma}\frac{\Gamma(-\gamma)}{\Gamma(1+\gamma)}\,+$$

$$+\,\psi(1)\,\frac{\Gamma(1+\gamma)}{\Gamma(-\gamma)}\,\frac{\partial}{\partial\,\gamma}\frac{\Gamma(-\gamma)}{\Gamma(1+\gamma)}\,+\,\left(\frac{\partial}{\partial\,\gamma}\frac{\Gamma(-\gamma)}{\Gamma(1+\gamma)}\right)\left(\frac{\partial}{\partial\,\gamma}\frac{\Gamma(1+\gamma)}{\Gamma(-\gamma)}\right)\,+\,\frac{\Gamma(1+\gamma)}{\Gamma(-\gamma)}\,\frac{\partial^2}{\partial\,f^2}\frac{\Gamma(-\gamma)}{\Gamma(1+\gamma)}\Big]\,\left(\frac{k}{\mu}\right)^{2\gamma}$$

and adding this expression to the Eq.10 we again obtain Eq.19 answer.

4 Green's function of the NLO BFKL equation

Now we consider a solution of NLO BFKL equation, i.e. Green's function of the equation constructed with the help of the found eigenfunctions. The Green's function of the Eq. (18) is

$$f(\omega,k_1,k_2)\,=\,\frac{1}{2\,\pi^2}\int d\gamma\,\left(\frac{k_1^2}{\mu^2}\right)^{\gamma}\,\left(\frac{k_2^2}{\mu^2}\right)^{\gamma^*}\,\frac{1}{\omega\,-\,\omega_{\gamma}^{LO}\,-\,\omega_{\gamma}^{NLO}} \tag{22}$$

for the case of conformal spin $n\,=\,0$. Coming back to the full anomalous dimension function of the eigenfunction from Eq. (11)

$$\gamma\,=\,f\,-\,\frac{\alpha_s\,\beta_0}{4\,\pi}\,, \tag{23}$$

we obtain

$$f(\omega,k_1,k_2)\,=\,\frac{1}{2\,\pi^2}\int d\,f\,\left(\frac{k_1^2}{\mu^2}\right)^{f-\,\frac{\alpha_s\,\beta_0}{4\,\pi}}\,\left(\frac{k_2^2}{\mu^2}\right)^{f^*-\,\frac{\alpha_s\,\beta_0}{4\,\pi}}\,\frac{1}{\omega\,-\,\omega_f} \tag{24}$$

where

$$\omega_f\,=\,\frac{N_c\,\alpha_s}{\pi}\left[\chi(-f)\,-\,\frac{\alpha_s\,\beta_0}{2\,\pi}\,\left(\frac{1}{2}\chi^2(-f)\,+\,\frac{1}{2}\,\psi^{'}(-f)\,-\frac{1}{2}\,\psi^{'}(1+f)\right)\right] \tag{25}$$

Redefining f as

$$f\,=\,-1/2\,-\,i\,\nu \tag{26}$$

we obtain our final expression for the Green's function of the equation

$$f(\omega,k_1,k_2)\,=\,\frac{\mu^2}{2\,\pi^2\,k_1\,k_2}\int_{-\infty}^{\infty}d\,\nu\,\left(\frac{k_2^2}{k_1^2}\right)^{i\,\nu}\,\frac{1}{\omega\,-\,\omega_{\nu}}\left(1\,-\,\frac{\alpha_s\,\beta_0}{4\,\pi}\,ln\left(\frac{k_1^2\,k_2^2}{\mu^4}\right)\right) \tag{27}$$

with

$$\omega_{\nu}\,=\,\frac{N_c\,\alpha_s}{\pi}\left[\chi(\nu\,)\,-\,\frac{\alpha_s\,\beta_0}{2\,\pi}\,\left(\frac{1}{2}\chi^2(\,\nu\,)\,-\,\frac{1}{2\,i}\,\psi^{'}(1/2\,+\,i\,\nu\,)\,+\,\frac{1}{2\,i}\,\psi^{'}(1/2\,-\,i\,\nu\,)\right)\right] \tag{28}$$

The diffusion approximation for the Green's function we obtain expanding ω_{ν} over ν

$$\omega_{\nu}\,=\,\omega_0\,-\,a^2\,\nu^2 \tag{29}$$

with

$$\omega_0\,=\,4\,\frac{N_c\,\alpha_s}{\pi}\,ln\,2\left[1\,-\,\frac{\alpha_s\,\beta_0}{\pi}\,\left(ln\,2\,-\,\frac{\pi^2}{16}\right)\right] \tag{30}$$

and

$$a^2\,=\,14\,\frac{N_c\,\alpha_s}{\pi}\,\zeta\,(3)\,\left(1\,-\,2\,\frac{\alpha_s\,\beta_0}{\pi}\,ln\,2\right)+\,\frac{N_c\alpha_s^2\,\beta_0\,\pi^2}{4} \tag{31}$$

Integration of the expression over ω and ν variables gives final answer for the Green's function in the diffusion approximation

$$F(s,k_1,k_2)\,\approx\,\frac{\mu^2}{2\,\pi\,a\,k_1\,k_2}\left(\frac{s}{s_0}\right)^{\omega_0}\,\frac{1}{\sqrt{\pi\,ln\,(\,s/s_0)}}\,exp\,\left(-\frac{ln^2\,(\,k_1^2\,/\,k_2^2\,)}{4\,a^2\,ln\,(\,s/s_0)}\right)\left(1-\,\frac{\alpha_s\,\beta_0}{4\,\pi}\,ln\left(\frac{k_1^2\,k_2^2}{\mu^4}\right)\right) \tag{32}$$

with the ω_0 and a from Eq. (30) and Eq. (31) correspondingly.

5 Conclusion

The shift of the anomalous dimension function of the LO eigenfunctions, represented by Eq. (11), is justified by the fact that in the NLO approximation the functions Eq. (5) are not eigenfunctions of the NLO equation. Therefore, transition from the Eq. (10) to the Eq. (12) does not affect on the correctness of the operator equation. Nevertheless, after a shift and redefinition of the NLO BFKL kernel we obtain equation Eq. (18) which is correctly defined as a operator equation with proper eigenfunctions and correspondingly conformal eigenvalue. From the formal point of view this shift reflects the functional structure of the γ in Eq. (5). Indeed, being function of α_s we can assume, that in the NLO approximation the LO γ acquires some corrections which are clarified in the shift $\gamma\,\rightarrow\,f$. Physically it means that together with NLO corrections of the kernel we need to account the NLO corrections of the anomalous dimension function of the eigenfunctions and this precisely that the shift Eq. (11) means. There is a strong assumption behind this statement, we assume that the functional form of the eigenfunctions does not change when NLO corrections of the kernel are considered and that all NLO corrections of the eigenfunctions are accumulated in the anomalous dimension function of the eigenfunction. This proposition could not be prooven directly and therefore we could consider the Eq. (18) as a effectively constructed operator equation where the request of the conformal eigenvalue determines a coefficient γ_1 in the Eq. (13) expansion.

The expansion Eq. (13) we can interpretate also as a perturbative expansion of the "full" eigenfunction in the case when we know only a part of NLO corrections to the kernel. In this case the redefinition Eq. (14) of the "shifted" or "full" eigenfunctions in the terms of initial "non-shifted" eigenfunctions looks like a renormalization group transition from one basis of eigenfunctions to another. As a results of this transition we have a new equation with the conformal eigenvalue and with the NLO corrections to the Green's function of the equation. Indeed, one of the main advantages of the proposed framework is the possibility to construct a Green's function of the equation. Using eigenfunctions of the equation the definition of the Green's function is standart and simple. Considering the Green's function of the equation, see Eq. (27), we obtain that the shift Eq. (11) leads to the simple form of the NLO corrections

to the LO Green's function and that the form of these corrections is depend on the coefficients γ_i in the expansion Eq. (13). Another advantage of the proposed framework is also the conformal structure of the eigenvalue which we consider as NLO Pomeron intercept and which could be written in the "diffusion" approximation, see Eq. (30) and Eq. (31).

Another interesting property of the proposed approach is that we considered dimensionless theory, introducing some external scale μ^2 even in LO approximation. The influence of this fact on the final result and the relation of proposed framework with the result of the NLO BFKL equation with the broken conformal invariance of the eigenvalue is a interesting subject which we plane investigate in our future studies.

Acknowledgments

S.B. especially grateful to N.Armesto, E.Levin and L.Lipatov for the usefull remarks about the subject of the note. This work was done with the support of the Ministerio de Educacion y Ciencia of Spain under project FPA2005-01963 together with Xunta de Galicia (Conselleria de Educacion).

References

[1] E. A. Kuraev, L. N. Lipatov and V. S. Fadin, Sov. Phys. JETP **45** (1977) 199 [Zh. Eksp. Teor. Fiz. **72** (1977) 377].

[2] I. I. Balitsky and L. N. Lipatov, Sov. J. Nucl. Phys. **28** (1978) 822 [Yad. Fiz. **28** (1978) 1597].

[3] V. S. Fadin and L. N. Lipatov, Phys. Lett. B **429** (1998) 127.

[4] G. Camici and M. Ciafaloni, Nucl. Phys. B **496** (1997) 305 [Erratum-ibid. B **607** (2001) 431].

[5] L. N. Lipatov, Sov. Phys. JETP **63** (1986) 904 [Zh. Eksp. Teor. Fiz. **90** (1986) 1536].

[6] L. N. Lipatov in Perturbative Quantum Chromodynamics, Ed. A. H. Mueller, World Scientific 1989.

[7] E. M. Levin and M. G. Ryskin, Phys. Rept. **189** (1990) 267.

[8] N. N. Nikolaev and B. G. Zakharov, Phys. Lett. B **327** (1994) 157.

[9] M. A. Braun, Phys. Lett. B **348** (1995) 190.

[10] E. Levin, Nucl. Phys. B **453** (1995) 303.

[11] Y. V. Kovchegov and A. H. Mueller, Phys. Lett. B **439** (1998) 428.

[12] N. Armesto, J. Bartels and M. A. Braun, Phys. Lett. B **442** (1998) 459.

[13] S. Bondarenko, E. Levin and C. I. Tan, Nucl. Phys. A **732** (2004) 73.

[14] V. S. Fadin and R. Fiore, Phys. Rev. D **72** (2005) 014018.

[15] V. S. Fadin, R. Fiore and A. Papa, Phys. Lett. B **647** (2007) 179.

[16] V. S. Fadin, R. Fiore, A. V. Grabovsky and A. Papa, Nucl. Phys. B **784** (2007) 49.

[17] Y. V. Kovchegov and H. Weigert, Nucl. Phys. A **789** (2007) 260.

[18] Y. V. Kovchegov and H. Weigert, Nucl. Phys. A **784** (2007) 188.

[19] I. Balitsky, Phys. Rev. D **75** (2007) 014001.

[20] I. Balitsky and G. A. Chirilli, Phys. Rev. D **77** (2008) 014019.

[21] A. H. Mueller, Nucl. Phys. B **415** (1994) 373.